**Chemistry 1S - Prof Paul May**

**Calculus Question for January Exam 2011**

1. Answer ***all*** parts (a) to (e)
2. Describe and sketch the main features of the function *u*(*x*) = cos(*ax*) where *a* is a constant. An accurate plot on graph-paper is *not* required.

*(5 marks)*

1. Describe and sketch the main features of the function *v*(*x*) = exp(–*bx*) where *b* is a constant. An accurate plot on graph-paper is *not* required.

*(5 marks)*

The function below, which is a product of the two functions in (a) and (b), is often used to represent damped simple-harmonic-motion:

*y*(*x*) = cos(*ax*) exp( –*bx*)

(c) Differentiate *y*(*x*) with respect to *x*.

*(6 marks)*

(d) Find a general expression involving *a* and *b* for the stationary points of *y*(*x*).

*(6 marks)*

(e) Sketch *y*(*x*) in your answerbooks for positive values of *x*, labelling the important parts of the curve. An accurate plot on graph-paper is *not* required.

*(8 marks)*

**Answers**

1. *u* = cos(*ax*) is a cosine function, *i.e*. a function which repeats itself with a period of 2π/*a*. It cycles between a maximum amplitude of +1 at *x*=0 and *x* = 2π/*a*, and a minimum amplitude of ‑1 at *x*=π/*a*. The function *u* = 0 at π/2*a* and 3π/2*a*.



1. *v* = exp( ‑*bx*) is an exponential decay function, with decay constant = *b*. It has a maximum value of 1 at *x*=0, and then drops away exponentially with increasing *x* at a rate determined by the magnitude of *b*. The function only reaches 0 at *x*=∞.



1. Product Rule: $\frac{dy}{dx}$ = cos(*ax*)( ‑*b*exp(-*bx*)) + (exp( ‑*bx*)( ‑*a*sin(*ax*)) [4 marks]

= exp( ‑*bx*) { ‑*b*cos(*ax*) – *a*sin(*ax*) } [2 marks for simplifying it]

1. Turning points occur when $\frac{dy}{dx}$ = 0, *i.e*. when

exp( ‑*bx*) { ‑*b*cos(*ax*) – *a*sin(*ax*) } = 0

This can occur if either exp( ‑*bx*) = 0, *i.e*. when *x* = ∞ (not a very useful solution)

or if ‑*b*cos(*ax*) – *a*sin(*ax*) = 0,

 –*a*sin(*ax*) = *b*cos(*ax*)

 tan(*ax*) = ‑*b* / *a*

*i.e.* for *x* values where *x* = (1/*a*) tan-1( ‑*b*/*a*)

1. *y*(*x*) will be a combination of both a cosine function and an exponential decay. The cosine function will be cyclic, as before, however instead of it oscillating between +1 and ‑1, the multiplying exponential function decreases these values at each cycle. Thus the curve is a ‘decaying cosine function’, with the period of oscillation given, as before by 2π/*a*, and the decay constant being *b*.

