1S Summer Exam June 2000 - Calculus Dr Paul May

1) Determine the following (1 mark each):

a) dy/dx if $y = 1000x^2$

b) dy/dr if $y = 5r^6 - r + 1$

c) dy/dx if $y = 3\cos x$

d) dp/dq if $p = 7e^{-q}$

2) Differentiate the following functions with respect to x, and simplify the result where possible:

a) $y = (x + 8)(1 - 3x)$

b) $y = 2x \ln x$

c) $y = \dfrac{5x}{(x-10)}$

d) $y = \sin(3x^2 + 5)$ (8 marks)

3) A function which is often used to represent the form of an electronic wavefunction in certain atoms is:

$$y = r^2 e^{-r}$$

a) This function has 3 stationary points. One is at $r = 0$, and another at $r =$ infinity. Differentiate this function and thence determine the coordinates (r,y) of the remaining stationary point. (6 marks)

b) The second differential of this function is:

$$\frac{d^2y}{dr^2} = e^{-r}\left(r^2 - 4r + 2\right)$$

Use this to determine whether the stationary point you just found is a local maximum or minimum. (2 Marks)

c) Hence sketch this function between $r = 0$ and $r =$ infinity. (4 marks)

Answers

1) [1mark for each].

a) $dy/dx = 2000x$ b) $dy/dr = 30r^5 - 1$
c) $dy/dx = -3\sin x$ d) $dp/dq = -7e^{-q}$

2) [2 marks each, 1 mark for differentiating correctly, 1 mark for simplifying it].

a) **Product Rule**: $dy/dx = (x + 8).(-3) + (1 - 3x).1$ = $-6x - 23$

b) **Product Rule**: $dy/dx = 2x.(1/x) + (\ln x).2$ = $2 + 2\ln x$

c) **Quotient Rule**: $dy/dx = \dfrac{(x-10).5 - 5x(1)}{(x-10)^2}$ = $-\dfrac{50}{(x-10)^2}$

d) **Function of a Function**: $dy/dx = \cos(3x^2 + 5).6x$ = $6x\cos(3x^2 + 5)$

3) a)
Product Rule: $-r^2e^{-r} + e^{-r}(2r)$ [2 mark] = $re^{-r}(2 - r)$ [1 mark]

For turning point, $re^{-r}(2 - r) = 0$, so either: $r = 0$ $\therefore$ $r = 0$,
$e^{-r} = 0$ $\therefore$ r = infinity
or $(2 - r) = 0$, $\therefore$ $\underline{r = 2}$
The last answer is the required one. [2 marks]

So the turning point is at (2, 0.54). [1 mark]

b) Determine the sign of the second differential, d^2y/dr^2. Putting in the value of $r = 2$, we get $d^2y/dr^2 =$ -0.27, which is **-ve**, so the t.p. is a local <u>maximum</u>. [2 marks]

c) Sketch: [4 marks]. Need to label axes correctly, get correct shape of graph, label t.p. correctly.