1S Summer exam 2001 - Calculus Dr Paul May

1) Determine the following:

a) dy/dx if $y = 3x^3$

b) dy/dm if $y = 4m^7 + 5m - 1$

c) $dy/d\theta$ if $y = 9\cos\theta$

d) dp/dq if $p = 1252e^{-56q}$ (6 marks)

2) Differentiate the following functions with respect to x, and simplify the result where possible:

a) $y = (5x + 2)(3 - 6x)$

b) $y = 2x^3 \ln x$

c) $y = \dfrac{8x}{(x+3)}$

d) $y = \sin(5x^4 - 9x)$ (8 marks)

3) The function :

$$y = \frac{2e^r}{r}$$

has stationary points at $r = \pm\infty$

a) Differentiate this function and thence determine the co-ordinates (r,y) of the remaining stationary point. (3 marks)

b) The second differential of this function is:

$$\frac{d^2y}{dr^2} = \frac{2e^r(r^2 - r + 2)}{r^3}$$

Determine whether the stationary point you just found is a local maximum or minimum. (3 Marks)

c) Hence sketch this function between $r = 0$ and $r = 8$. (4 marks)

Answers

1) [1mark for (a) and (b), 2 marks for the rest].

a) $dy/dx = 9x^2$ b) $dy/dr = 28m^6 + 5$
c) $dy/d\theta = -9\sin x$ d) $dp/dq = -70112e^{-56q}$

2) [2 marks each].

a) Product Rule: $(5x + 2).(-6) + (3 - 6x).5 = 3 - 60x$

b) Product Rule: $2x^3(1/x) + (\ln x).6x^2 = 2x^2(1 + 3\ln x)$

c) Quotient Rule: $\dfrac{(x+3).8-8x(1)}{(x+3)^2} = \dfrac{24}{(x+3)^2}$

d) Funct. of a Funct.: $\cos(5x^4 - 9x).(20x^3 - 9) = (20x^3 - 9)\cos(5x^4 - 9x)$

3)
a)
Quotient Rule: $dy/dr = (r.2e^r - 2e^r.1) / r^2 = 2e^r(r - 1) / r^2$ [2 marks]

For turning point, $2e^r(r - 1) / r^2 = 0$, so either: $r^2 = \infty \Rightarrow r = \pm\infty$,
$2e^r = 0 \Rightarrow r = -\infty$
or $(r - 1) = 0, \Rightarrow$ **$r = 1$**

The last answer is the required one.

So the turning point is at (1, 5.44). [1 mark]

b) Determine the sign of the second differential, d^2y/dr^2. Putting in the value of $r = 1$, we get $d^2y/dr^2 = 4e$, which is **+ve**, so the t.p. is a local minimum. [3 marks]

c) Sketch: (must get correct shape, label axes, and indicate t.p., and infinities for full 4 marks).