1S Summer exam 2003 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d).

   Determine the following:

   (a) dy/dx if $y = 5x^9$

   (1 mark)

   (b) dk/dp if $k = 3p^6 + 17p - 6$

   (1 mark)

   (c) $d\beta/d\theta$ if $\beta = -5\cos\theta$

   (2 marks)

   (d) dj/dm if $j = 73e^{-25m}$

   (2 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to x, and simplify the result where possible:

   (a) $y = (3x + 2)(1 - 11x)$

   (b) $y = -55x^{12} \ln x$

   (c) $y = \dfrac{(11x+7)}{(3x-2)}$

   (d) $y = \tan(x^6 - 3x^2)$

   (8 marks)

3. Answer ***all*** parts (a) to (c).

   Consider the function: $y(x) = 5\sin(2x + \pi/2)$, where x is in radians.

   (a) Differentiate this function, and thence determine the co-ordinates (x,y) of the stationary points and whether they are local maxima or minima.

   (5 Marks)

   (b) Hence sketch this function between $x = 0$ and $x = \pi$.

   (3 marks)

   (c) Calculate the area under this function between $x = 0$ and $x = \pi/4$.
   [Hint: the relationship $\sin(-m + \pi/2) = \cos m$ might be useful].

   (2 marks)

Answers

1) [1mark for (a) and (b), 2 marks for the rest].

a) $dy/dx = 45x^8$ b) $dk/dp = 18p^5 + 17$

c) $d\beta/d\theta = +5\sin\theta$ d) $dj/dm = -1825e^{-25m}$

2) [2 marks each].

a) Product Rule: $(3x + 2).(-11) + (1 - 11x).3 = -19 - 66x$

b) Product Rule: $-55x^{12}(1/x) + (\ln x).(-660\, x^{11}) = -55x^{11}(1 + 12 \ln x)$

c) Quotient Rule: $\dfrac{(3x-2).11-(11x+7)(3)}{(3x-2)^2} = \dfrac{-43}{(3x-2)^2}$

d) Funct. of a Funct.: $[1 / \{\cos^2(x^6 - 3x^2)\}] \times (6x^5 - 6x) = \dfrac{6x(x^4-1)}{\cos^2(x^6-3x^2)}$

3)

a) Using Func. Of Func. Rule, $dy/dx = 5\cos(2x + \pi/2) \times 2 = \underline{10\cos(2x + \pi/2)}$ [2 marks]

At the t.p. $dy/dx = 0$, so $10\cos(2x + \pi/2) = 0$,
so $\cos(2x + \pi/2) = 0$.
Since $\cos^{-1}(0) = \pi/2, 3\pi/2, 5\pi/2,$... *etc*, then:
$(2x + \pi/2) = \pi/2, 3\pi/2, 5\pi/2,$ *etc*.

When $(2x + \pi/2) = \pi/2$, $2x = 0$, so $x = 0$, and $y = 5$, $\Rightarrow$ a t.p at (0, 5)
or when $(2x + \pi/2) = 3\pi/2$, $2x = \pi$, so $x = \pi/2$, and $y = -5$, $\Rightarrow$ a t.p at $(\pi/2, -5)$
and when $(2x + \pi/2) = 5\pi/2$, $2x = 2\pi$, so $x = \pi$, and $y = -5$, $\Rightarrow$ a t.p at $(\pi, 5)$
...*etc*.

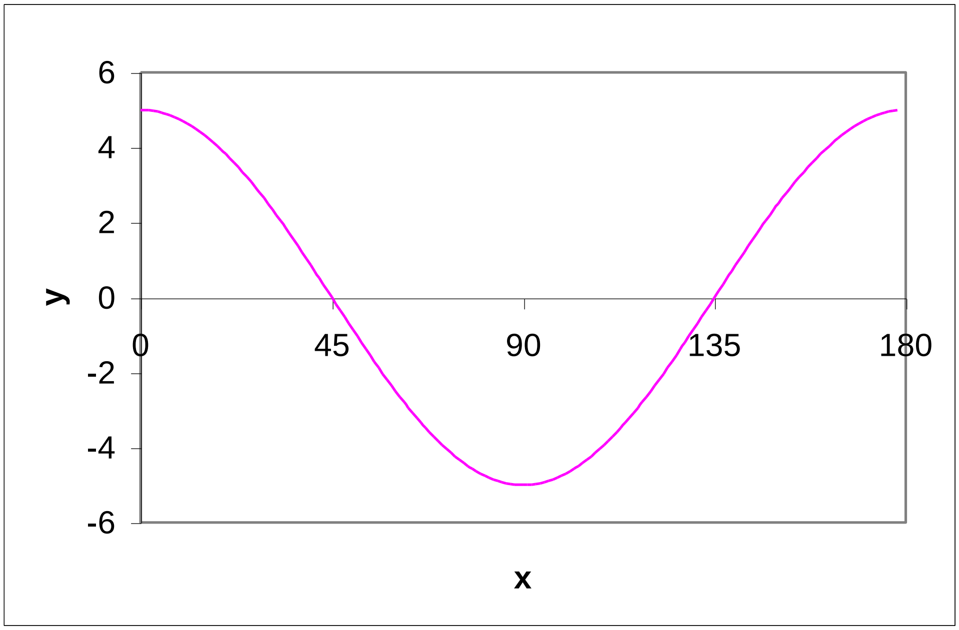
[2 marks]

b) $d^2y/dx^2 = \underline{-20\sin(2x + \pi/2)}$, and at $x = 0$ this has a value of -20. So the t.p. is a <u>maximum</u>!

At $x = \pi/2$, d^2y/dx^2 has a value of +20, so it's a <u>minimum.</u>
At $x = 3\pi/2$, d^2y/dx^2 has a value of -20, so it's a <u>maximum</u>.

So it's an oscillating function (as you might expect for a sine wave), with repeating maxima and minima. [3 marks]

c) The sine function will take values from +1 to –1, so y will oscillate from +5 to –5. The x multiplier is 2, so the function will oscillate with twice the frequency of a normal sine wave, *i.e.* one wavelength every π rather than every 2π.



note: add t.p labels, etc...

d) We cannot integrate the original function easily, but using the relationship $\sin(-m + \pi/2) = \cos m$ we can see that $5\sin(2x + \pi/2) = 5\cos(-2x)$, which is much easier to integrate:

$$\text{Area} = \int_0^{\pi/4} 5\cos(-2x).dx \qquad = \left[-\frac{5}{2}\sin(-2x)\right]_0^{\pi/4}$$

= 5/2 – 0 = <u>2.5 sq. units</u>