1S Summer exam 2004 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Determine the following:

   (a) dy/dx if $y = 7x^5$

   (b) dk/dp if $k = 6p^5 + 21p - 8$

   (c) $d\beta/d\theta$ if $\beta = 9\tan\theta$

   (d) dj/dm if $j = 21e^{-12m}$

   (4 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to x, and simplify the result where possible:

   (a) $y = (5x + 1)(7 - 3x)$

   (b) $y = 33x^5 \ln x$

   (c) $y = \dfrac{(6x+5)}{(6x^3-2)}$

   (d) $y = \cos(x^5 - 3x^4)$

   (8 marks)

3) A function which is often used to represent the form of an electronic wavefunction in certain atoms is:

$$y = r^2 e^{-r}$$

a) This function has 3 stationary points. One is at $r = 0$, and another at $r =$ infinity. Differentiate this function and thence determine the coordinates (r, y) of the remaining stationary point. *(4 marks)*

b) Differentiate this function again, determine whether the stationary point you just found is a local maximum or minimum. *(5 Marks)*

c) Hence sketch this function between $r = 0$ and $r = 8$. *(3 marks)*

Answers

1)

a) $dy/dx = 35x^4$

b) $dk/dp = 30p^4 + 21$

c) $d\beta/d\theta = +9/\cos^2\theta$

d) $dj/dm = -252e^{-12m}$

2)

a) Product Rule: $(5x + 1).(-3) + (7 - 3x).5 \quad = \quad 32 - 30x$

b) Product Rule: $33x^5(1/x) + (\ln x.165\, x^4) \quad = \quad 33x^4 (1 + 5 \ln x)$

c) Quotient Rule: $\dfrac{(6x^3 - 2).6 - (6x + 5)(18x^2)}{(6x^3 - 2)^2} \quad = \quad \dfrac{-72x^3 - 90x^2 - 12}{(6x^3 - 2)^2}$

d) Funct. of a Funct.: $-\sin (x^5 - 3x^4) \times (5x^4 - 12x^3) \quad = \quad -(5x^4 - 12x^3) \sin (x^5 - 3x^4)$

3) a)

Product Rule: $-r^2e^{-r} + e^{-r}(2r)$ [2 mark] $= re^{-r}(2 - r)$

For turning point, $re^{-r}(2 - r) = 0$, so either:

$r = 0 \quad \therefore \quad r = 0,$

$e^{-r} = 0 \quad \therefore \quad r = \text{infinity}$

or $(2 - r) = 0, \quad \therefore \quad \underline{r = 2}$

The last answer is the required one.

So the turning point is at (2, 0.54).

b) This is quite tricky:

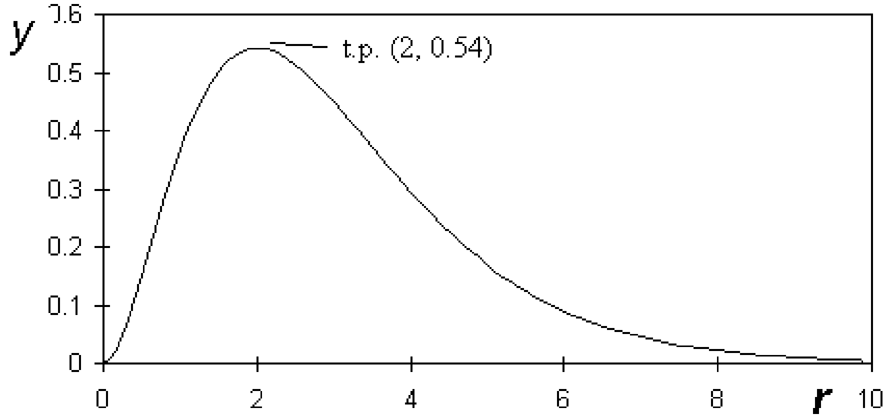
$$d^2y/dx^2 = re^{-r}(-1) + (2-r)\,[\, r(-e^{-r}) + e^{-r}(1)]$$

$$= -re^{-r} - 2re^{-r} + 2e^{-r} + r^2e^{-r} - re^{-r}$$

$$\underline{= re^{-r}\,(r^2 - 4r + 2)}$$

Determine the sign of the second differential, d^2y/dr^2. Putting in the value of $r = 2$, we get $d^2y/dr^2 =$ -0.27, which is **-ve**, so the t.p. is a local <u>maximum</u>.

c) Sketch. Need to label axes correctly, get correct shape of graph, label t.p. correctly.


y
0.6
0.5
0.4
0.3
0.2
0.1
0
t.p. (2, 0.54)
0
2
4
6
8
r
10