1S Summer exam 2007 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Determine the following:

   (a) dy/dx if $y = 270x^{67}$

   (b) du/dg if $u = 1.62g^3 + 5g - 3\times10^6$

   (c) dΩ/dΞ if $\Omega = 3\tan\,\Xi$

   (d) d🕷/d🐈 if 🕷 = 9 exp(–7🐈) + 6🐈

   (4 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to x, and simplify the result where appropriate:

   (a) $y = \dfrac{1}{x^3} - \sqrt[5]{x^4} + \dfrac{3}{\sqrt{x}}$

   (b) $y = 9e^{6x}\cos x$

   (c) $y = \dfrac{3(x^5 - 2x)}{(x^3 - 2x + 100)}$

   (d) $y = 6\ln\left(-\dfrac{1}{3x}\right)$

   (8 marks)

3. Answer ***all*** parts (a) to (c).

The function $y = \dfrac{3e^r}{r}$ has a stationary point at $r = -\infty$

a) Differentiate this function and thence determine the co-ordinates (r,y) of the remaining stationary point.

(3 marks)

b) The second differential of this function is: $\dfrac{\mathrm{d}^2y}{\mathrm{d}r^2} = \dfrac{3e^r(r^2 - 2r + 2)}{r^3}$

Determine whether the stationary point you just found is a local maximum or minimum.

(3 Marks)

c) Hence sketch this function between $r = 0$ and $r = 4$.

(6 marks)

Answers

1)

a) $dy/dx = 18090x^{66}$ b) $du/dg = 4.86g^2 + 5$

c) $d\Omega/d\Xi = 3 / \cos^2\Xi$ d) d[symbol]/d[symbol] = -63 exp(-7[symbol]) + 6

2)

a) Rules for Indices: $y = x^{-3} - x^{4/5} + 3x^{-1/2}$

$$dy/dx = -3x^{-4} - (4/5)x^{-1/5} - (3/2)x^{-3/2}$$
$$= -\frac{3}{x^4} - \frac{4}{5\sqrt[5]{x}} - \frac{3}{2\sqrt{x^3}}$$

b) Product Rule: $9e^{6x}\,(-\sin x) + (\cos x)\,54e^{6x} = 9e^{6x}(6\cos x - \sin x)$

c) Quotient Rule:

$$\frac{dy}{dx} = \frac{(x^3 - 2x + 100)3(5x^4 - 2) - 3(x^5 - 2x)(3x^2 - 2)}{(x^3 - 2x + 100)^2} = \frac{3\{(x^3 - 2x + 100)(5x^4 - 2) - (x^5 - 2x)(3x^2 - 2)\}}{(x^3 - 2x + 100)^2}$$

d) Funct. of a Funct.: $dy/dx = \dfrac{6}{-\dfrac{1}{3x}} \times \left(\dfrac{1}{3x^2}\right) = -18x \times \left(\dfrac{1}{3x^2}\right) = -\dfrac{6}{x}$

Alternatively, by the Laws of Logs,

$6\ln\left(-\dfrac{1}{3x}\right)$ is the same as $-6\ln(-3x)$, so $dy/dx = \dfrac{-6}{-3x} \times -3 = -\dfrac{6}{x}$

3) (a) Quotient Rule: $dy/dr = (r.3e^r - 3e^r.1) / r^2 = 3e^r(r - 1) / r^2$

For turning point, $3e^r(r - 1) / r^2 = 0$, so either: $r^2 = \infty \Rightarrow r = \infty$,

or $3e^r = 0 \Rightarrow r = -\infty$

or $(r - 1) = 0, \Rightarrow$ ***r* = 1**

The last answer is the required one.

When $r = 1$, $y = 3e^1/1 = 8.15$. So the turning point is at (1, 8.15).

b) Putting in the value of $r = 1$, we get $d^2y/dr^2 = 3e$, which is **+ve**, so the t.p. is a local minimum.

c) Sketch: (must get correct shape, label axes, and indicate t.p. for full 6 marks).

When $r = 0$, $y = \infty$.

When r = large, the e^r term makes $y = \infty$

When r = a small number (*e.g.* 1/100), the r in the denominator makes y very large.