1S Summer exam 2008 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Determine the following:

   (a) dy/dx if $y = 16x^{36}$

   (b) du/dg if $u = 1.7g^3 + (3\times10^6)g$

   (c) $d\Omega/d\psi$ if $\Omega = 3\tan\psi$

   (d) d✈/d👄 if ✈ = 2 exp(–3👄) - 5👄

   (4 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to x, and simplify the result where appropriate:

   (a) $y = \dfrac{1}{2x^3} - \sqrt[3]{x^2} + \dfrac{4}{\sqrt{x}}$

   (b) $y = 12e^{-2x}\sin x$

   (c) $y = \dfrac{2(3x^5 - x)}{(3x^3 - 2)}$

   (d) $y = 5\ln\left(\dfrac{1}{2x^2}\right)$

   (8 marks)

3. Answer ***all*** parts (a) to (c).

   Consider the quartic function: $y = (x - 1)(x + 1)(x - 3)(x + 3)$

   (a) Multiply out the brackets in this equation.

   (2 marks)

   (b) Differentiate either version of this equation and hence find the co-ordinates of the stationary points.

   (*4 marks*)

   (c) Find the second differential of this equation, and hence find whether each of the stationary points is a local maximum, minima or point of inflection.

   (*3 marks*)

   (d) Sketch this function between x = -4 and x = +4.

   (*3 marks*)

Answers

1)

a) $dy/dx = 576x^{35}$

b) $du/dg = 5.1g^2 + 3\times10^6$

c) $d\Omega/d\psi = 3 / \cos^2\psi$

d) d✈/d👄 = -6 exp(-3👄) - 5

2)

a) Rules for Indices: $y = ½x^{-3} – x^{2/3} + 4x^{-1/2}$

$$dy/dx = -(3/2)x^{-4} – (2/3)x^{-1/3} – 2x^{-3/2}$$

$$= -\frac{3}{2x^4} - \frac{2}{3\sqrt[3]{x}} - \frac{2}{\sqrt[2]{x^3}}$$

b) Product Rule: $12e^{-2x}$ (cos x) + (sin x) ($-24e^{-2x}$) = $12e^{-2x}$ (cos x - 2sin x)

c) Quotient Rule:

$$\frac{dy}{dx} = \frac{(3x^3-2)\times 2(15x^3-1) - 2(3x^5-x)\times(9x^2)}{(3x^3-2)^2} = \frac{(3x^3-2)(30x^3-2) - 18x^3(3x^4-1)}{(3x^3-2)^2}$$

d) Funct. of a Funct.: $dy/dx = 5\times 2x^2 \times\left(\frac{-2}{2x^3}\right) = 10x^2\times\left(\frac{-1}{x^3}\right) = -\frac{10}{x}$

Alternatively, by the Laws of Logs,

$5\ \ln\left(\frac{1}{2x^2}\right)$ is the same as $-5\ \ln(2x^2)$, so $dy/dx = \frac{-5}{2x^2}\times 4x = -\frac{10}{x}$

3) (a) $y = (x - 1)(x + 1)(x - 3)(x + 3) \quad = \quad x^4 - 10x^2 + 9$

(b) $dy/dx = 4x^3 - 20x$

At the tps, $dy/dx = 0$, so

$$4x^3 - 20x = 0$$
$$x^3 - 5x = 0$$
$$x(x^2 - 5) = 0$$

Therefore, $x = 0$ or $(x^2 - 5) = 0$, *i.e.* $x = \pm\sqrt{5}$

When $x=0$, $y = +9$,
When $x=+\sqrt{5}$, $y = -16$,
When $x= -\sqrt{5}$, $y = -16$.

So there are ***three*** turning points, at: $(0, 9)$, $(\sqrt{5}, -16)$, $(-\sqrt{5}, -16)$

(c) $d^2y/dx^2 = 12x^2 - 20$

When $x=0$, $d^2y/dx^2 = -20$ (*i.e.* negative), so that the tp at $(0, 9)$ is a <u>maximum</u>.
When $x=+\sqrt{5}$, $d^2y/dx^2 = +40$ (*i.e.* positive), so that the tp at $(\sqrt{5}, -16)$ is a <u>minimum</u>.
When $x= -\sqrt{5}$, $d^2y/dx^2 = +40$ (*i.e.* positive), so that the tp at $(-\sqrt{5}, -16)$ is a <u>minimum</u>.

(d) Need to sketch graph, get correct shape, label axes properly, and label the turning points and places where it crosses the axes to get full marks.