1S Summer exam 2009 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Determine the following:

   (a) dy/dx if $y = 5x^{26}$

   (b) du/dh if $u = 2.3h^3 + (3\times10^{-6})h$

   (c) $d\Omega/d\psi$ if $\Omega = 7 \tan \psi$

   (d) d♬/d👂 if ♬ = 2 exp(–4👂) - 3👂

   (4 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to *x*, and simplify the result where appropriate:

   (a) $y = \dfrac{1}{2x^3} + \dfrac{3}{\sqrt[4]{x^5}}$

   (b) $y = 6e^{-3x} \cos x$

   (c) $y = \dfrac{(x^5 - 3x)}{(3x^3 - 1)}$

   (d) $y = 3 \ln\left(\dfrac{x^3}{5}\right)$

   (8 marks)

3. Answer ***all*** parts (a) to (c).

   Consider the function: $y = (x - 1)^3 (5x - 1)$

   (a) Differentiate this equation and hence find the co-ordinates of the stationary point(s).

   (*4 marks*)

   (b) Find the second differential of this equation, and hence find whether the stationary point(s) is/are a local maximum, minima or points of inflection.

   (*4 marks*)

   (c) Sketch the function between $x = 0$ and $x = +3$.

   (*4 marks*)

Answers

1)

a) $dy/dx = 130x^{25}$ b) $du/dh = 6.9h^2 + 3\times10^{-6}$

c) $d\Omega/d\psi = 7 / \cos^2\psi$ d) d♬/d👂 = -8 exp(-4👂) - 3

2)

a) Rules for Indices: $y = \frac{1}{2}x^{-3} + 3x^{-5/4}$

$$dy/dx = -(3/2)x^{-4} - (15/4)x^{-9/4}$$

$$= -\frac{3}{2x^4} - \frac{15}{4\sqrt[4]{x^9}}$$

b) Product Rule: $6e^{-3x}(-\sin x) + (\cos x)(-18e^{-3x}) = -6e^{-3x}(3\cos x + \sin x)$

c) Quotient Rule:

$$\frac{dy}{dx} = \frac{(3x^3-1)(5x^4-3)-(x^5-3x)(9x^2)}{(3x^3-1)^2}$$

d) Funct. of a Funct.: $dy/dx = 3 \times (5/x^3) \times 3x^2/5 = 9/x$

3) (a) $y = (x-1)^3(5x-1)$

Most students multiplied out the brackets and then differentiated it, but this is a bad idea since the multiplication is tricky and takes many lines, so many students made mistakes in this. But also the function you get is a quartic polynomial, which is easy to differentiate but which is almost impossible to factorise to find the turning points.

Use Product Rule, plus F-of-F Rule and it only takes 2 lines:

$$\frac{dy}{dx} = (x-1)^3(5) + (5x-1)\times3(x-1)^2\times(1)$$

$$\frac{dy}{dx} = 5(x-1)^3 + 3(x-1)^2(5x-1)$$

And, factorising:

$$\frac{dy}{dx} = (x-1)^2\{(5(x-1) + 3(5x-1)\}$$

$$= (x-1)^2\{(5x-5) + 15x-3)\}$$

$$= \mathbf{(x-1)^2(20x-8)}$$

At t.p. $\frac{dy}{dx} = 0$, so $(x-1)^2(20x-8) = 0$,

So either $(x-1)^2 = 0$, which means $x = 1$ and therefore $y = 0$,

Or $(20x-8) = 0$, which means $x = 8/20$ (0.4) and $y = -0.216$.

So there are only 2 t.p.s, at **(1, 0) and (0.4, -0.216)**

[For a quartic function you'd expect 3 t.p.s, so if we only got two, either we did it wrong or this is a clue that one of the t.p.s might be 'special'].

(b) $d^2y/dx^2 = (x-1)^2\,(20) + (20x-8).2(x-1).(1)$

$$= \mathbf{20(x-1)^2 + (40x-16)(x-1)}$$

(i) When $x = 1$, $d^2y/dx^2 = 0$, so this is a point of inflection.
(ii) When $x = 0.4$, $d^2y/dx^2 = 7.2$, *i.e.* +ve, so it's a minimum.

(c) Need to sketch graph, get correct shape, label axes properly, and label the turning points and places where it crosses the axes to get full marks.

From original eqn:
When $x = 0$, $y = +1$.
When $x = 1$, $y = 0$ (the p.o.i)
When $x = 0.4$, $y = -0.216$, minimum t.p.
When x = large and +ve, y = tends to $5x^4$, *i.e.* also large and positive