1S Summer exam 2010 - Calculus Dr Paul May

1. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Determine the following:

   (a) dy/dx if $y = 3x^{101}$

   (b) du/dh if $u = 3.7h^4 + (3\times10^6)h^2$

   (c) dΩ/dψ if Ω = 5 cos ψ

   (d) d👌/d👓 if 👓 = 2 exp(–7👌) - 5👌^{2}

   (4 marks)

2. Answer ***all*** parts (a) to (d). All parts carry equal marks.

   Differentiate the following functions with respect to *x*, and simplify the result where appropriate:

   (a) $y = -\dfrac{6}{\sqrt[3]{x^5}}$

   (b) $y = 7e^{-2x} \sin x$

   (c) $y = \dfrac{(2x^5 - 3)}{(x^2 - 1)}$

   (d) $y = \ln\left(\dfrac{3x^3}{4}\right)^2$

   (8 marks)

3. Answer ***all*** parts (a) to (c).

Consider the function: $y = \left(\dfrac{x-1}{x+1}\right)^2$

(a) Differentiate this equation and hence find the co-ordinates of the stationary point(s).

(*4 marks*)

(b) Find the second differential of this equation, and hence find whether the stationary point(s) is/are a local maximum, minima or points of inflection.

(*4 marks*)

(c) Sketch the function between $x = -5$ and $x = +5$.

(*4 marks*)

Answers

1)

a) $dy/dx = 303x^{100}$

b) $du/dh = 14.8h^3 + (6\times10^6)h$

c) $d\Omega/d\psi = -5 \sin \psi$

d) d👌/d👓 = -14 exp(-7👌) - 10👌

2)

a) Rules for Indices: $y = -6x^{-5/3}$

$$dy/dx = +(30/3)x^{-8/3} = \frac{10}{\sqrt[3]{x^8}}$$

b) Product Rule: $7e^{-2x}(\cos x) + (\sin x)(-14e^{-2x}) = 7e^{-2x}(\cos x - 2\sin x)$

c) Quotient Rule: $\frac{dy}{dx} =$

$$\frac{(x^2-1)(10x^4)-(2x^5-3)(2x)}{(x^2-1)^2} = \frac{(10x^6-10x^4)-(4x^6-6x)}{(x^2-1)^2} = \frac{(6x^6-10x^4+6x)}{(x^2-1)^2} = \frac{2x(3x^5-5x^3+3)}{(x^2-1)^2}$$

d) Funct. of a Funct.: $dy/dx = \left(\frac{4}{(3x^3)}\right)^2 \times 2\left(\frac{3x^3}{4}\right) \times \frac{9x^3}{4} = 6/x$

alternatively, rewrite it as $y = 2\ln\left(\frac{3x^3}{4}\right)$

So that $dy/dx = 2 \times \left(\frac{4}{(3x^3)}\right)\times\frac{9x^2}{4} = \frac{6}{x}$

3) (a) $y = \left(\frac{x-1}{x+1}\right)^2$

Use F-of-F Rule and then the Quotient Rule:

$$\frac{dy}{dx} = 2\left(\frac{x-1}{x+1}\right)\times\frac{(x+1).1-(x-1).1}{(x+1)^2}$$

$$\frac{dy}{dx} = \frac{4(x-1)}{(x+1)^3}$$

At t.p. $\frac{dy}{dx}=0$, so either $4(x-1) = 0$ so that $x = +1$, and $y = 0$,

or $\frac{1}{(x+1)^3} = 0$, so that $x = \infty$ and $y = -1$. This is an unusual solution and indicates something odd happens at this point.

So there are only 2 t.p.s, at **(1, 0) and (∞, -1)**

(b) $$d^2y/dx^2 = \frac{(x+1)^3 4 - 4(x-1)3(x+1)^2.1}{(x+1)^6} = \frac{4(x+1)-12(x-1)}{(x+1)^4}$$

(i) When $x = 1$, $d^2y/dx^2 = 1/2$, *i.e.* +ve so this is a <u>minimum</u>.
(ii) When $x = \infty$, d^2y/dx^2 is undefined….

(c) Need to sketch graph, get correct shape, label axes properly, and label the turning points and places where it crosses the axes to get full marks.

From original eqn:
When $x = 0$, $y = +1$.
When $x = 1$, $y = 0$ (the p.o.i)
When $x = -1$ $y = \infty$; so have an asymptote at $x = -1$
When x = large and +ve, y = tends to +1
When x = large and -ve, y = tends to +1