Scattering . 1 **Basics**

1.1 Recommended Books:

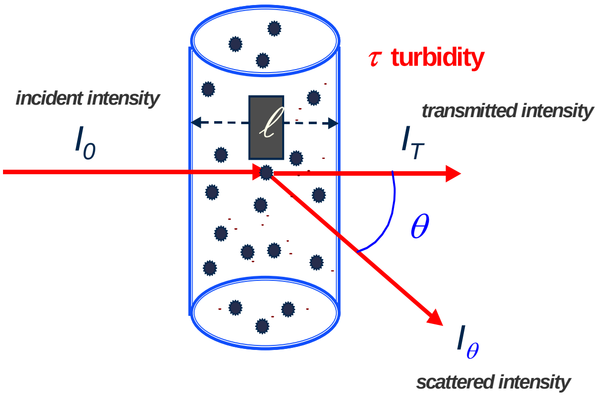
1] B.Chu Laser *Light Scattering* 2nd Ed. Academic Press 1991
2] R. J. Hunter *Foundations of Colloid Science 2nd Ed* Oxford 2001
3] H. R.Cruyt *Colloid Science Vol. I* Elsevier 1952
4] M. Kerker *The scattering of light and other electromagnetic radiation* Academic Press 1969.
5] R. Richards *Scattering Methods in Polymer Science* Ellis Horwood 1995

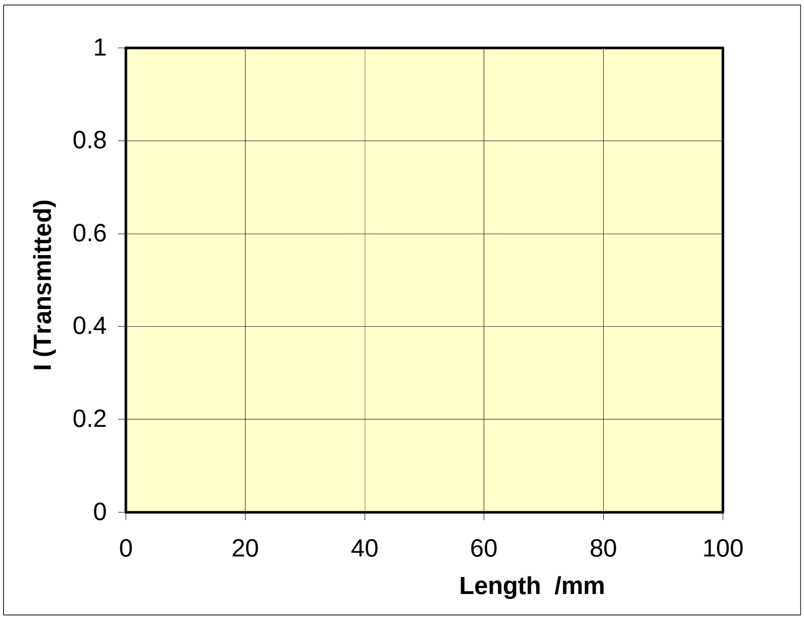
Classical papers

6] Rayleigh L. *Nature* **3** 234 [1871] 7] Debye P., *Ann. Physik.* **30,** 57 [1909]
8] Mie G., *Ann. Physik. [Leipzig]* **25,** 377 [1908]
9] A, Guinier X-ray diffraction In Crystals (1963) etc. Dover [1994]

1.2 Definitons:

Light is absorbed, transmitted or scattered: $\lambda \approx d$ diffraction
$\lambda << d$ refraction (d particle diameter)
I_θ scattered intensity at an angle θ;
I_0 incident; I_T transmitted $\tau = I_T / I_0$ [1.1}
Beer Lambert Law: $I_T = I_o \exp(-\varepsilon c \ell)$ [1.2]
c/concentration , ε/extinction coefficient.





1.3

Elastic scattering

Only changes in momentum transfer vector $\overline{Q}$: energy[λ] is fixed: *n=1*

$\bar{Q} = \bar{k}_o - \bar{k}_\theta \qquad k_o = k_\theta = 2\pi / \lambda$

$Q^2 = (\bar{k}_o - \bar{k}_o)^2 = 2k_o^2 - 2\bar{k}_o\bar{k}_o = 2k_o^2(1-\cos(\theta))$ $\qquad Q = \dfrac{4\pi}{\lambda}\sin(\theta/2) \qquad [1.3]$

$Q^2 = 2k_o^2(2\sin^2(\theta/2))$

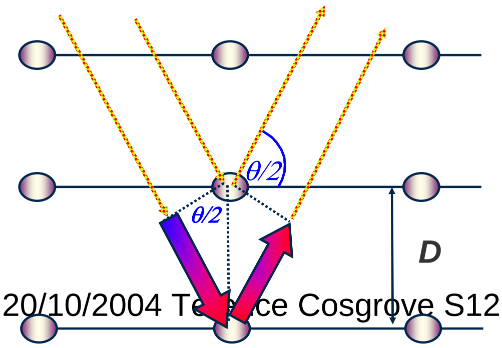
Also $c = \nu\lambda$ $v = f\lambda$ [1.4} Inelastic $k_0 \neq k_\theta$ quasielastic $k_0 \approx k_\theta$

1.4 Radiation types:

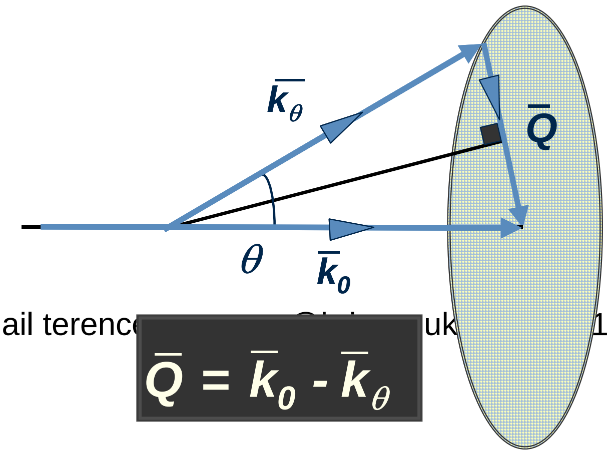
Sound	v = 330 $m\ s^{-1}$	f = 1Hz – 30 kHz	λ = 0.1m - 330m
Light	c = 3.0 x 10 8 $m\ s^{-1}$	ν = 7 x1014 - 4 x 1014 Hz	λ = 434 - 768 nm
Neutrons	v = 4000 - 400 $m\ s$-1	m_N = 1.675E-27 kg :	λ = 0.1 - 1 nm
X-rays	c = 3.0 x 10^8 $m\ s^{-1}$	ν = 3 x10^{18} - 3 x 10^{17} Hz	λ = 0.1 - 1 nm

1.5 Bragg's Law

$n\lambda = 2D\sin(\theta/2)$ [1.5] : Interference when phase difference is an integer number of wavelengths as in figure below.



path difference = 2Dsin($\theta/2$)=nλ



1.6 Rayleigh Scattering

The conditions for Raleigh scattering (1871) from individual particles diameter d is that $d < \lambda/20$ and refractive index, $n \approx 1$

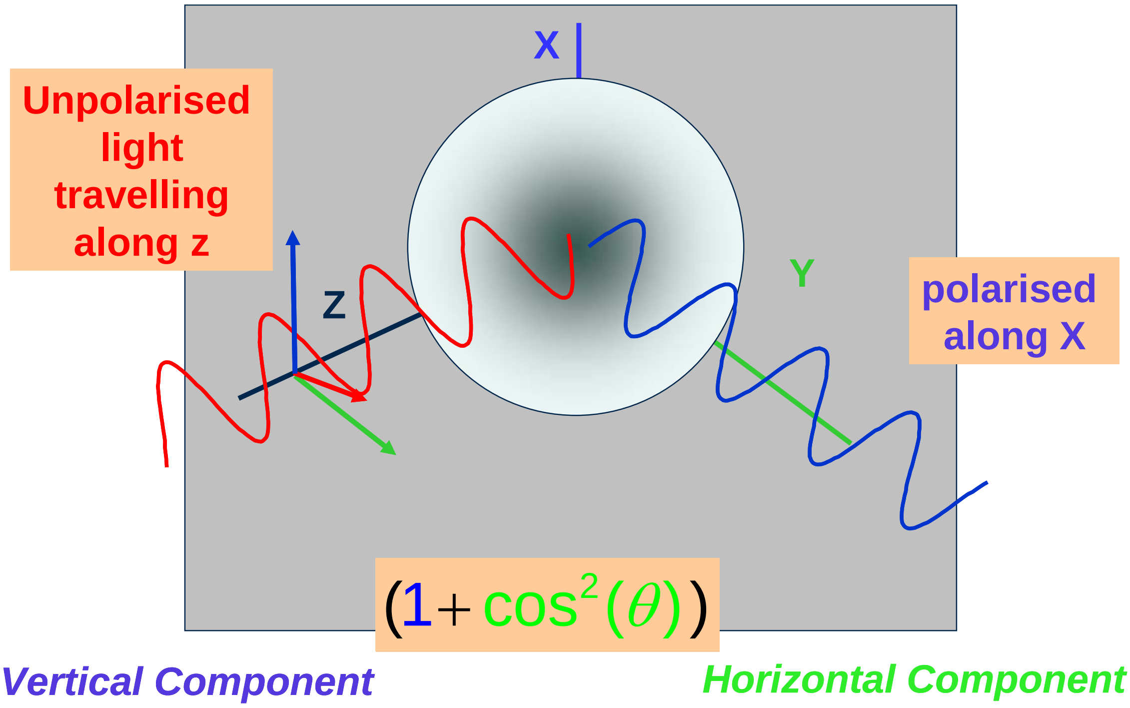
(a) Polarisation of light.

$I_o = I_{Horizontal} + I_{Vertical.}$ [1.6]

Unpolarised light travels along *z* (see figure). The polarisation can broken up into a horizontal (y) component and vertical (x) component. The scattered wave also has a polarisation components along x and y but viewed along the y axis (the propagation direction) there is only one component remaining the x vertical component which is still normal to the propagation direction. Scattering depends on the polarisability α .

$$\mu = \alpha E \quad [1.7]$$

μ dipole moment, *E* Electric field strength. α is large if the HOMO-LUMO energy gap is small.



(b) Dimensional analysis:

Scattering of light is due to electrons so $I \sim f(\alpha)$. It also must depend on the wavelength of the light, λ and the distance of the observer from the scattering body and the relative refractive indices $m=n_p/n_o$ of the particle *(p)* and the medium *(o)*

$$I_\theta = I_0.f(\alpha, r, \lambda, n_p, n_o, \theta) \qquad [1.8]$$

When light falls on a particle it induces a dipole moment which is proportional to the polarisability, α. The scattering field, $\bar{E}$, is then proportional to α. .Classically α is described by the Lorentz-Lorentz equation when there is no permanent dipole*:*

$$\alpha = 4\pi\varepsilon_0 a^3(m^2 - 1)/(m^2 + 2) \quad [1.9]$$

a is the radius, $m=n_p/n_o$ and ε_o is the vacuum permittivity.
So $\bar{E}$ is proportional to volume a^3*, so* the Intensity which is proportional to $E^2 \propto \alpha^2 \propto a^6$. The Inverse square law shows us that $I \propto r^{-2}$

So combining these gives $I_\theta \propto I_0.a^6 r^{-2} f(\lambda, n_p, n_o, \theta)$ [1.10]

I_θ / I_0 is dimensionless as are the values of n so this implies that $I_\theta \propto \lambda^{-4}$ [1.11]
Hence .

$$I_\theta \propto \frac{I_0.a^6}{r^2\lambda^4} f(n_p, n_o, \theta) \quad [1.12]$$

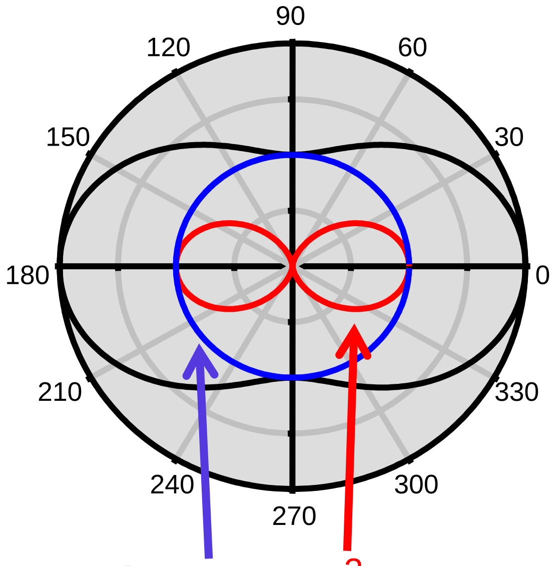
So the intensity of the Rayleigh scatterer depends on frequency ν^4 and this explains many astronomical observations- why the sky is blue , the green flash etc.

(c) Angular dependence.
The polarisation as we have seen in angle dependent and at 90^0 to I_o the light is completely plan polarised. The intensity must also be positive and a function which gives this is $(1 + \cos^2(\theta))$ where '1' is the vertical component and $\cos^2(\theta)$ *is the horizontal component. S*o combining this with [1.12] and [1.9] gives the full Rayleigh expression:

$$\frac{I_\theta}{I_0} = \frac{9\pi^2(n_p^2 - n_o^2)^2}{2r^2\lambda^4(n_p^2 + 2n_o^2)^2} V_p^2 N_p (1 + \cos^2(\theta)) \quad [1.13]$$

where V_p is the particle volume and N_p is the number of particles per unit volume

NB	Distance	$\sim 1/r^2$
	Frequency	$\sim 1/\lambda^4 \sim \nu^4$
	Size	$\sim V_p^2 \sim a^6$
	Concentration	$\sim N_p$
	Refractive index difference	$\sim \Delta(n^2)^2$



$$I = k(1 + \cos^2(\theta))$$

Vertical Component ***Horizontal Component***