1S Summer exam 2002 - Calculus Dr Paul May

1. Answer **all** parts (a) to (d).

    Determine the following:

    (a) dy/dx if $y = 8x^4$

    (1 mark)

    (b) dy/dp if $y = 6p^5 + 3p - 2$

    (1 mark)

    (c) $d\beta/d\theta$ if $y = -7\cos\theta$

    (2 marks)

    (d) dj/dk if $j = 253e^{-27k}$

    (2 marks)

2. Answer **all** parts (a) to (d). All parts carry equal marks.

    Differentiate the following functions with respect to x, and simplify the result where possible:

    (a) $y = (7x + 2)(3 - 8x)$ (b) $y = -5x^7 \ln x$

    (c) $y = \dfrac{6x}{(3x - 2)}$ (d) $y = \sin(5x^6 - 2x^2)$

    (8 marks)

3. Answer **all** parts (a) to (c).

    Consider the function $y(x) = (x + 2)^3$.

    (a) Differentiate this function (without multiplying out the brackets) and thence determine the co-ordinates (x,y) of the stationary point(s).

    (4 marks)

    (b) Do the stationary point(s) correspond to local maxima, minima, or point(s) of inflection?

    (4 Marks)

    (c) Hence sketch this function between $x = -4$ and $x = +1$.

    (3 marks)

Answers

1) [1mark for (a) and (b), 2 marks for the rest].

a) $dy/dx = 32x^3$

b) $dy/dp = 30p^4 + 3$

c) $d\beta/d\theta = +7\sin\theta$

d) $dj/dk = -6831e^{-27k}$

2) [2 marks each].

a) Product Rule: $(7x + 2).(-8) + (3 - 8x).7$ $=$ $5 - 112x$

b) Product Rule: $-5x^7(1/x) + (\ln x).(-35x^6)$ $=$ $-5x^6(1 + 7\ln x)$

c) Quotient Rule: $\dfrac{(3x - 2).6 - 6x(3)}{(3x - 2)^2}$ $=$ $\dfrac{-12}{(3x - 2)^2}$

d) Funct. of a Funct.: $\cos(5x^6 - 2x^2).(30x^5 - 4x)$ $=$ $(30x^5 - 4x)\cos(5x^6 - 2x^2)$

3)
a) Using Func. Of Func. Rule, $dy/dx = 3(x + 2)^2.(1) =$ $3(x + 2)^2$ [2 marks]

At the t.p. $dy/dx = 0$, so $3(x + 2)^2 = 0$, so there's only 1 soln at $x = -2$.
Feeding $x = -2$ back into the original eqn, we get that the t.p is at: (-2, 0).   [2 marks]

b) $d^2y/dx^2 = 6(x + 2)$, and at $x = -2$ this has a value of 0. So the t.p. is a point of inflexion!
Checking the gradient either side of the t.p.: at $x = -3$, $dy/dx = +3$, so it's increasing.
At $x = -1$, $dy/dx = +3$, so it's increasing too.   [3 marks]

c) When x is very large and +ve, y is v. large and +ve. When x is large and –ve, y is large
and –ve. When $x = -2$, $y = 0$, and this is the t.p. When $x = 0$, the intercept $y = 8$.

$y = (x+2)^3$