

1S Summer exam 2005 - Calculus Dr Paul May

1. Answer **all** parts (a) to (d). All parts carry equal marks.

Determine the following:

- (a) dy/dx if $y = 9x^3$
- (b) dk/dp if $k = 3p^2 + 2p - 5$
- (c) $d\beta/d\theta$ if $\beta = 7\tan \theta$
- (d) dj/dm if $j = 10e^{-190m}$

(4 marks)

2. Answer **all** parts (a) to (d). All parts carry equal marks.

Differentiate the following functions with respect to x , and simplify the result where possible:

- (a) $y = (7x + 5)(10 - 5x)$
- (b) $y = 4x^{21} \ln x$
- (c) $y = \frac{(3x^2 + 5x)}{(6x^3 - 2x + 3)}$
- (d) $y = 5\sin (x^6 - 2x^7)$

(8 marks)

3. Answer **all** parts (a) to (c).

Consider the function $y(x) = (x + 3)^3$.

- (a) Differentiate this function (without multiplying out the brackets) and thence determine the co-ordinates (x,y) of the stationary point(s).
(4 marks)
- (b) Do the stationary point(s) correspond to local maxima, minima, or point(s) of inflection?
(4 Marks)
- (c) Hence sketch this function between $x = -5$ and $x = +1$.
(4 marks)

Answers

1)

a) $dy/dx = 27x^2$

b) $dk/dp = 6p + 2$

c) $d\beta/d\theta = +7/\cos^2\theta$

d) $dj/dm = -1900e^{-190m}$

2)

a) Product Rule: $(7x + 5).(-5) + (10 - 5x).7 = 45 - 70x$

b) Product Rule: $4x^{21}(1/x) + (\ln x. 84 x^{20}) = 4x^{20}(1 + 21 \ln x)$

c) Quotient Rule:
$$\frac{(6x^3 - 2x + 3).(6x + 5) - (3x^2 + 5x)(18x^2 - 2)}{(6x^3 - 2x + 3)^2} =$$
$$\frac{-18x^4 - 60x^3 - 6x^2 + 18x - 15}{(6x^3 - 2x + 3)^2}$$

d) Funct. of a Funct.: $5\cos(x^6 - 2x^7) \times (6x^5 - 14x^6) = 5(6x^5 - 14x^6) \cos(x^6 - 2x^7)$

3) a)

Using Func. of Func. Rule, $dy/dx = 3(x + 3)^2.(1) = 3(x + 3)^2$ [2 marks]

At the t.p. $dy/dx = 0$, so $3(x + 3)^2 = 0$, so there's only 1 soln at $x = -3$.

Feeding $x = -3$ back into the original eqn, we get that the t.p is at: $(-3, 0)$. [2 marks]

b) $d^2y/dx^2 = 6(x + 3)$, and at $x = -3$ this has a value of 0. So the t.p. is a point of inflexion!

Checking the gradient either side of the t.p.: at $x = -4$, $dy/dx = +3$, so it's increasing.

At $x = -2$, $dy/dx = +3$, so it's increasing too. [4 marks]

c) When x is very large and +ve, y is v. large and +ve. When x is large and -ve, y is large and -ve. When $x = -3$, $y = 0$, and this is the t.p. When $x = 0$, the intercept $y = 27$.

Students must label axes, t.p. and intercept on graph. [4 marks]

